Mathematics Induction Assignment

A Level Maths builds on the skills you developed in GCSE Maths, with a particular focus on algebra. It's important that we can move on quickly to the new content, so we don't want to spend a lot of time at the start of the year revising GCSE – that's going to be your job.

Your task is to make sure you are ready to go in September. We will check this by giving you a test in the second week, which will be very similar to the attached assignment.

This assignment is a selection of questions which cover skills you will need to be fluent in to make a successful start to A Level Maths. You should be familiar with all of them, although some of them may need more practice or support. The answers are given – so you can mark the work yourself. We expect you to check your work, and to show that you have marked it.

We expect you to come to the first lesson with your completed assignment, ready to ask for help on any specific questions you didn't manage to do. We will take in the assignment – not to mark it (as you will have done that already), but so we can look at how you set your work out. We expect you to show full workings for all the questions (not just a single answer).

As well as making sure that you are ready mathematically to start the course, this assignment also introduces you to the style of learning you can expect in AS Maths – working independently, checking your own work, and using lessons to get detailed help where you are stuck.

Working with others is strongly encouraged. It's not important whether you can do all these questions immediately by yourself, or if you spend a lot of time revising and getting help. What matters is that you can do them for yourself in September, so if you don't get the answers straightaway, it would be a good idea to practise similar questions.

If you need help completing this assignment, or more questions to practise

• Use your GCSE notes, revision guides and past papers

• Use the notes in this document

• Use online resources, for example

- MyMaths
- Hegarty Maths
- Mr Barton
- Khan Academy
- http://www.mathcentre.ac.uk/resources/uploaded/2009algebrarefresherv4.pdf (you do not need to be able to do "partial fractions")
- http://www.cimt.org.uk/projects/mepres/step-up/index.htm
- http://bit.ly/2rQcQHU (OCR Bridging the Gap)

OCR Core 1 GCSE revision

Exercise level 1

Do not use a calculator in this exercise.

1. Simplify the following expressions:
   (i) $2x + 3y - x + 5y + 4x$
   (ii) $5a - 2b + 3c - 2a + 5b$
   (iii) $4p + q - 6p - 5q + 5p + 4q$

2. Multiply out the brackets and simplify where possible:
   (i) $3(2x + 3y)$
   (ii) $4(3a - 2b) - 3(a + 2b)$
   (iii) $p(2p - q) + 2q(p - 3q)$

3. Multiply out these expressions.
   (i) $(x + 1)(x - 3)$
   (ii) $(x + 2)(2x + 1)$
   (iii) $(x - 3)(x - 4)$
   (iv) $(3x + 2)(x - 2)$
   (v) $(2x + 1)(4x - 1)$
   (vi) $(1 - 2x)(1 + x)$
   (vii) $(3 + 2x)(x - 1)$
   (viii) $(5x - 3)(2x + 5)$

4. Factorise the following expressions:
   (i) $10ab + 5ac$
   (ii) $2x^2 + 4xy - 8xz$
   (iii) $3s^2t - 9s^3t + 12s^2t^2$

5. Simplify the following as much as possible:
   (i) $\frac{2a^2b}{4ab^2}$
   (ii) $\frac{12p^2qr^3}{9pq^2r}$
   (iii) $\frac{x^2y + xy^2}{x + y}$
   (iv) $\frac{a}{2b} \times \frac{3bc}{a^2} \times \frac{a}{6c}$

6. Factorise:
   (i) $3xy + xy^2$
   (ii) $4a^3b + 2a^2b^2 + a^4b^2$
   (iii) $2x^2 - xy + 2xy - 6y^2$

7. Simplify:
   (i) $\frac{a^2 - b^2}{a^3b - a^2b^2}$
   (ii) $\frac{ax - 3ay}{(x + y)(x - 3y)}$

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Exercise level 2

Do not use a calculator in this exercise.

1. Write as single fractions:

(i) $\frac{2x}{5}+\frac{3x}{2}$ (ii) $\frac{3a}{4}-\frac{2b}{3}$

(iii) $\frac{2x+1}{12}-\frac{x-2}{8}$ (iv) $\frac{3x+4}{2x}-\frac{5x+6}{3x}$

(v) $\frac{1}{p}+\frac{1}{q}$ (vi) $\frac{a}{2b}+\frac{5b}{3a}$

2. Solve the following equations:

(i) $2x - 3 = 8$ (ii) $3y + 2 = y - 5$

(iii) $3 - 2a = 3a - 1$ (iv) $3(p - 3) = 2(2p + 1)$

(v) $2(1 - z) + 3(z + 3) = 4z + 1$ (vi) $\frac{2b+1}{5}=\frac{3-b}{4}$

3. Simplify:

(i) $\frac{2(x+2)}{3}-\frac{2}{x}$ (ii) $\frac{(x+y)}{a}-\frac{(x-y)}{b}$

(iii) $\frac{1}{x}+\frac{x-1}{2}-\frac{x-3}{3}$ (iv) $\frac{xy}{z}-\frac{yz}{x}$

4. The largest angle of a triangle is three times as big as the smallest angle.
The third angle is 20° greater than the smallest angle.
Find all three angles of the triangle.

5. In a restaurant, there are 24 tables, some of which seat four people, and the rest seat 6 people. The restaurant can hold 114 people altogether.
How many tables seat four people?

6. Michelle is doing a Statistics project on the heights of students in her class.
She has written:
    Mean height of boys = 165 cm
    Mean height of girls = 159 cm
    Mean height of whole class = 162.2 cm
There are 30 students in Michelle's class.
How many boys and how many girls are there?

7. Make x the subject of each of these formulae:

(i) $ax+b=c$ (ii) $p-qx^2=r$

(iii) $\sqrt{\frac{x}{s}}=t$ (iv) $a-\frac{b}{x}=c$

(v) $px+q=a-bx$ (vi) $y=\frac{1}{w(z-x^2)}$

OCR Core 1 GCSE revision

Notes and Examples

These notes contain subsections on

You will have met most, if not all, of this work at GCSE. To succeed at AS level you must master these skills fluently so that you can carry them out quickly and accurately, almost without thinking.

Collecting like terms

There are three different types of "like term" in this expression. There are terms in a, terms in b, and a term in ab. Notice that the term in ab cannot be combined with either the terms in a or the terms in b, but remains as a term on its own.

Example 1

Simplify the expression

$$3a + 2b - a + 3b - 2ab + 2a$$

Solution

$$\begin{aligned} 3a + 2b - a + 3b - 2ab + 2a &= 3a - a + 2a + 2b + 3b - 2ab \\ &= 4a + 5b - 2ab \end{aligned}$$

In the example the expression has been rewritten with each set of like terms grouped together, before simplifying by adding / subtracting the like terms. You may well not need to write down this intermediate stage.

For practice in examples like this one, try the interactive questions ***Collecting terms***.

Multiplying out brackets

Example 2

Simplify the expressions

(i) $3(p - 2q) + 2(3p + q)$

(ii) $2x(x + 3y) - y(2x - 5y)$

Solution

Each term in the bracket must be multiplied by the number or expression outside the bracket.

(i) $$\begin{aligned} 3(p - 2q) + 2(3p + q) &= 3p - 6q + 6p + 2q \\ &= 9p - 4q \end{aligned}$$

(ii) $2x(x + 3y) - y(2x - 5y) = 2x^2 + 6xy - 2xy + 5y^2$
$= 2x^2 + 4xy + 5y^2$

Multiplying out two brackets of the form $(ax + b)(cx + d)$ gives a quadratic function. Each term in the first bracket must be multiplied by each term in the second bracket.

Example 3
Multiply out $(x+2)(3x-4)$

Solution

$$(x+2)(3x-4) = 3x^2 - 4x + 6x - 8$$
$$= 3x^2 + 2x - 8$$

You need to multiply each term in the first bracket by each term in the second. Use FOIL – First, Outer, Inner, Last.

You can see a step-by-step version of this example in this ***PowerPoint presentation***.

You can test yourself on expanding brackets using the Flash resource ***Expanding brackets***.

There is also a ***video*** which shows multiplying out brackets, starting from an investigation that you might have seen at GCSE. It then looks at multiplying out first just one bracket, then two brackets. It then goes on to deal with some harder examples, which are beyond the requirements of this section but are well worth looking at. The whole video lasts 40 minutes, so you may wish to fast-forward over parts of it if your time is limited.

Factorising

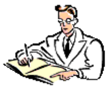
To factorise an expression, look for numbers and/or letters which are common factors of each term. We often talk about "taking out a factor" – this can cause confusion as it tends to make you think that subtraction is involved. In fact you are, of course, dividing each term by the common factor which you are "taking out".

Example 4
Factorise the following expressions.
(i) $6a + 12b + 3c$
(ii) $6x^2y - 10xy^2 + 2xy$

Solution
(i) 3 is a factor of each term.
$6a + 12b + 3c = 3(2a + 4b + c)$

(ii) $2xy$ is a factor of each term.
$6x^2y - 10xy^2 + 2xy = 2xy(3x - 5y + 1)$

Check your answers by multiplying out the brackets.



You may also like to try the ***Algebra Puzzle***, either on your own or in a group, in which you need to be able to recognise equivalent algebraic expressions and involves multiplying out brackets, factorising and simplifying.

Adding and subtracting algebraic fractions

Algebraic fractions follow the same rules as numerical fractions. When adding or subtracting, you need to find the common denominator, which may be a number or an algebraic expression.

Example 5
Simplify

(i) $\frac{2x}{3}+\frac{x}{4}-\frac{5x}{6}$

(ii) $\frac{1}{2x}-\frac{1}{x^2}$

Solution

(i) The common denominator is 12, as 3, 4 and 6 are all factors of 12.

$$\frac{2x}{3}+\frac{x}{4}-\frac{5x}{6}=\frac{8x}{12}+\frac{3x}{12}-\frac{10x}{12}$$
$$=\frac{8x+3x-10x}{12}$$
$$=\frac{x}{12}$$

(ii) The common denominator is $2x^2$.

$$\frac{1}{2x}-\frac{1}{x^2}=\frac{x}{2x^2}-\frac{2}{2x^2}$$
$$=\frac{x-2}{2x^2}$$

Simplifying fractions

You are familiar with the idea of "cancelling" to simplify numerical fractions: for example, $\frac{9}{12}$ can be simplified to $\frac{3}{4}$ by dividing both the numerator and the denominator by 3. You can also cancel before carrying out a multiplication, to make the numbers simpler:

e.g. $\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{9}^{3}}{\cancel{4}_{2}}=\frac{3}{2}$. The same technique can be used in algebra. As with factorising, remember that "cancelling" involves dividing, not subtracting.

Example 6
Simplify

(i) $\dfrac{6xy^3 + 2x^2y}{10x^2y}$

(ii) $\dfrac{3a}{a+1} \times \dfrac{2a+2}{a+2}$

Solution

(i) It is very important to remember that you can only "cancel" if you can divide each term in both the numerator and denominator by the same expression. In this case, don't be tempted to divide by $2x^2y$ – although this is a factor of both $2x^2y$ and $10x^2y$, it is not a factor of $6xy^3$. In a case like this, it may be best to factorise the top first, so that it is easier to see the factors.

$$\frac{6xy^3 + 2x^2y}{10x^2y} = \frac{2xy(3y^2 + x)}{10x^2y} = \frac{3y^2 + x}{5x}$$

$2xy$ is a common factor of both top and bottom

(ii) Again, factorise where possible first.

$$\frac{3a}{a+1} \times \frac{2a+2}{a+2} = \frac{3a}{\cancel{a+1}} \times \frac{2(\cancel{a+1})}{a+2} = \frac{6a}{a+2}$$

$(a + 1)$ is a common factor of both top and

Notice that you cannot cancel a here, as it is not a factor of $a + 2$.

You may also find the Mathcentre video ***Simplifying algebraic fractions*** useful.

Linear equations

A linear equation involves only terms in x (or whatever variable is being used) and numbers. So it has no terms involving x^2, x^3 etc. Equations like these are called linear because the graph of an expression involving only terms in x and numbers (e.g. $y = 2x + 1$) is always a straight line.

Solving a linear equation may involve simple algebraic techniques such as gathering like terms and multiplying out brackets. Example 7 shows a variety of techniques that you might need to use.

Example 7
Solve these equations.

(i) $5x - 2 = 3x + 8$

(ii) $3(2y - 1) = 4 - 2(y - 3)$

(iii) $\dfrac{2a-1}{3} = 2a+3$

Solution

(i)
$$5x - 2 = 3x + 8$$
Add 2 to each side
$$5x = 3x + 8 + 2$$
$$5x = 3x + 10$$
Subtract $3x$ from each side
$$5x - 3x = 10$$
$$2x = 10$$
Divide each side by 2
$$x = 5$$

(ii)
$$3(2y-1) = 4 - 2(y-3)$$
Multiply out the brackets
$$6y - 3 = 4 - 2y + 6$$
$$6y - 3 = 10 - 2y$$
Add 3 to each side
$$6y = 13 - 2y$$
Add $2y$ to each side
$$8y = 13$$
Divide each side by 8
$$y = \tfrac{13}{8}$$

(iii)
$$\frac{2a-1}{3} = 2a+3$$
Multiply both sides by 3
$$2a - 1 = 3(2a+3)$$
Multiply out the brackets
$$2a - 1 = 6a + 9$$
Add 1 to each side
$$2a = 6a + 10$$
Subtract $6a$ from each side
$$-4a = 10$$
Divide both sides by -4
$$a = -2.5$$

You can see more examples of solving linear equations using the Flash resource ***Linear equations***.

You can also look at a ***video*** which demonstrates the solution of a wide range of linear equations.

In Example 8, the problem is given in words and you need to express this algebraically before solving the equation.

Example 8

Jamila has a choice of 2 tariffs for text messages on her mobile phone.

Tariff A: 10p for the first 5 messages each day, 2p for all others

Tariff B: 4p per message

How many messages would Jamila need to send each day for the two tariffs to cost the same? (She always sends at least 5!)

Solution
Let the number of messages Jamila sends per day be n.
Under Tariff A, she has to pay 10p for each of 5 messages and 2p for each of n - 5 messages.
Cost = $50 + 2(n - 5)$
Under Tariff B, she has to pay 4p for each of n messages.
Cost = $4n$

For the cost to be the same

$$50 + 2(n - 5) = 4n$$
$$50 + 2n - 10 = 4n$$
$$40 + 2n = 4n$$
$$40 = 2n$$
$$20 = n$$

She needs to send 20 messages per day for the two tariffs to cost the same.

For practice in examples like this one, try the interactive questions ***Forming and solving linear equations***.

Changing the subject of a formula

Changing the subject of a formula is similar to solving an equation, but you are working with letters rather than numbers.

Example 9
The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.
Make r the subject of this formula.

Solution

$$V = \pi r^2 h$$

Divide both sides by πh

$$\frac{V}{\pi h} = r^2$$

Square root both sides

$$\sqrt{\frac{V}{\pi h}} = r$$

Finish by writing the equation with r on the left side.

$$r = \sqrt{\frac{V}{\pi h}}$$

In the next example, two different methods are shown. The answers look a bit different but they are equivalent. Make sure that you can see how you could rewrite one solution to give the other.

Example 10

The surface area A of a cylinder with radius r and height h is given by $A = 2\pi r(h+r)$.
Make h the subject of this formula.

Solution (1)

$A = 2\pi r(h+r)$ — Multiply out the brackets

$A = 2\pi rh + 2\pi r^2$ — Subtract $2\pi r^2$ from each side

$A - 2\pi r^2 = 2\pi rh$ — Divide each side by $2\pi r$

$\frac{A - 2\pi r^2}{2\pi r} = h$ — Rewrite with h on the left

$h = \frac{A - 2\pi r^2}{2\pi r}$

Solution (2)

$A = 2\pi r(h+r)$ — Divide each side by $2\pi r$

$\frac{A}{2\pi r} = h + r$ — Subtract r from each side

$\frac{A}{2\pi r} - r = h$ — Rewrite with h on the left side

$h = \frac{A}{2\pi r} - r$

In the next example, the new subject appears more than once. You need to collect the terms involving the new subject together and then factorise to isolate the new subject.

Example 11

Make x the subject of the formula $cx - a = a(b+x)$.

Solution

$cx - a = a(b+x)$ — Multiply out the brackets

$cx - a = ab + ax$ — Subtract ax from each side to ... ether

$cx - ax - a = ab$ — Add a to each side

$cx - ax = ab + a$ — Factorise

$(c-a)x = a(b+1)$ — Divide both sides by $c - a$

$x = \frac{a(b+1)}{c-a}$

There is a ***video*** showing a number of examples of rearranging formulae.

Summer Induction Work Answers

Ex L1

1. i) $5x + 8y$

ii) $3a + 3b + 3c$

iii) $3p$

2. i) $6x + 9y$

ii) $9a - 14b$

iii) $2p^2 + pq - 6q^2$

3. i) $x^2 - 2x - 3$

ii) $2x^2 + 5x + 2$

iii) $x^2 - 7x + 12$

iv) $3x^2 - 4x - 4$

v) $8x^2 + 2x - 1$

vi) $1 - x - 2x^2$

vii) $2x^2 + x - 3$

viii) $10x^2 + 19x - 15$

4. i) $5a(2b + c)$

ii) $2x(x + 2y - 4z)$

iii) $3s^2t(1 - 3s + 4t)$

5. i) $\frac{a}{2b}$

ii) $\frac{4pr^2}{3q}$

iii) xy

iv) $\frac{1}{4}$

6. i) $xy(3 + y)$

ii) $a^2b(4a + 2b + a^2b)$

iii) $(2x - 3y)(x + 2y)$

7. i) $\frac{(a+b)}{a^2b}$

ii) $\frac{a}{x+y}$

Ex L2

1. i) $\frac{19x}{10}$

ii) $\frac{9a-8b}{12}$

iii) $\frac{x+8}{24}$

iv) $\frac{-1}{6}$

v) $\frac{q+p}{pq}$

vi) $\frac{3a^2+10b^2}{6ab}$

2. i) $x = \frac{11}{2}$

ii) $y = \frac{-7}{2}$

iii) $a = \frac{4}{5}$

iv) $p = -11$

v) $p = \frac{10}{3}$

vi) $b = \frac{11}{13}$

3. i) $\frac{2(x+3)(x-1)}{3x}$

ii) $\frac{(b-a)x+(b+a)y}{ab}$

iii) $\frac{x^2+3x+6}{6x}$

iv) $\frac{y(x-z)(x+z)}{xz}$

4. 32°, 56°, 96°

5. 15 tables which seat 4 people

6. 16 boys and 14 girls

7. i) $x = \frac{c-b}{a}$

ii) $x = \sqrt{\frac{p-r}{q}}$

iii) $x = st^2$

iv) $x = \frac{b}{a-c}$

v) $x = \frac{a-q}{p+b}$

vi) $x = \sqrt{z - \frac{1}{wy}}$